Sol1 Given matrix $A = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$

① Interchanging $R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix}$$

② $R_2 \rightarrow R_2 - R_1$

$$\begin{bmatrix} 3 & -9 & 12 & -9 & 6 & 15 \\ 0 & 2 & -4 & 4 & 2 & -6 \\ 0 & 3 & -6 & 6 & 4 & -5 \end{bmatrix}$$

③ $R_3 \rightarrow R_3 - \frac{3}{2} R_2$

$$\begin{bmatrix} \boxed{3} & -9 & 12 & -9 & 6 & 15 \\ 0 & \boxed{2} & -4 & 4 & 2 & -6 \\ 0 & 0 & 0 & 0 & \boxed{1} & 4 \end{bmatrix}$$

We have the echelon form and pivot columns marked.

Now furthur reducing it into reduced echelon form

$R_2 \rightarrow \frac{1}{2} R_2$

$$\begin{bmatrix} 3 & -9 & 12 & -9 & 0 & -9 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$R_1 \rightarrow R_1 + 9R_2$

$$\begin{bmatrix} 3 & 0 & -6 & 9 & 0 & -72 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

$$R_1 \longrightarrow \frac{1}{3} R_1$$

$$\begin{bmatrix} 1 & 0 & -2 & 3 & 0 & -24 \\ 0 & 1 & -2 & 2 & 0 & -7 \\ 0 & 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

This is the reduced echelon form of the original matrix.

(2) Given matrix is

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 3 & 6 & 7 \end{bmatrix}$$

$R_3 \to \cancel{R_3} \; R_3 - \frac{3}{2}R_2$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 2 & 4 & 6 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

$R_2 \to \frac{1}{2}R_2$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

$R_3 \to -\frac{1}{2}R_3$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \to R_2 - 3R_3, \quad R_1 \to R_1 - 2R_3$

$$\begin{bmatrix} 1 & 2 & -3 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_1 \rightarrow R_1 + 3R_2$

$$\begin{bmatrix} 1 & 2 & 0 & 7 & 0 \\ 0 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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(3) $$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

To find the dimension of the row space and column space, first we will find the row reduced echlon form of the given matrix.

So, after applying $R_2 \rightarrow R_2 - 2R_1$

$R_3 \rightarrow R_3 - 2R_1$

$R_4 \rightarrow R_4 + R_1$

$$\begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 1 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_3 \rightarrow R_3 - R_2$

$$\begin{bmatrix} \boxed{1} & -3 & 4 & -2 & 5 & 4 \\ 0 & 0 & \boxed{1} & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 & \boxed{1} & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 4R_2$

$$\begin{bmatrix} 1 & -3 & 0 & -14 & 13 & 28 \\ 0 & 0 & 1 & 3 & -2 & -6 \\ 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \to R_1 - 13R_3$

$R_2 \to R_2 + 2R_3$

$$\begin{bmatrix} \boxed{1} & -3 & 0 & -14 & 0 & -37 \\ 0 & 0 & \boxed{1} & 3 & 0 & 4 \\ 0 & 0 & 0 & 0 & \boxed{1} & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence, the row reduced echelon form have three linearly independent rows and columns.

So, dim. of row space = 3

and dimension of column space = 3

Sol^n 4

(a) $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

Null space of $A = \{ (x, y, z, w) : A(x, y, z, w)^t = 0 \}$

$$\Rightarrow \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

$$\Rightarrow \quad x + 4y + 5z + 2w = 0$$

$$2x + y + 3z = 0$$

$$-x + 3y + 2z + 2w = 0$$

we get $x = \frac{2}{7}w - z$

$y = -\frac{4}{7}w - z$

$\therefore$ Null space $= \left\{ \left(\frac{2}{7}w - z, -\frac{4}{7}w - z, z, w \right) : w, z \in \mathbb{R} \right\}$

$= \text{Span} \left\{ \left(\frac{2}{7}, -\frac{4}{7}, 0, 1 \right), (-1, -1, 1, 0) \right\}.$

$\therefore$ Basis of Null space

$= \left\{ \left(\frac{2}{7}, -\frac{4}{7}, 0, 1 \right), (-1, -1, 1, 0) \right\}.$

(b) We have column 1 and column 2 are linearly independent and columns 3 and 4 are dependent on 1 and 2

Hence basis for column space $= \left\{ \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \\ 3 \end{pmatrix} \right\}.$

(c)

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

operation 1 :- $R_2 \longrightarrow R_2 - 2R_1$

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & -7 & -7 & -4 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

operation 2 :- $R_3 \longrightarrow R_3 + R_1$

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & -7 & -7 & -4 \\ 0 & 7 & 7 & 4 \end{bmatrix}$$

3 :- $R_2 \longrightarrow R_2\left(-\frac{1}{7}\right)$

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 1 & 4/7 \\ 0 & 7 & 7 & 4 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 7R_2$

$$\begin{bmatrix} 1 & 4 & 5 & 2 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 \rightarrow R_1 - 4R_2$

$$\begin{bmatrix} 1 & 0 & 1 & -2/7 \\ 0 & 1 & 1 & 4/7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

So basis of row space by reducing the matrix to row reduced echelon form is

$$\left\{ \left(1, 0, 1, -\frac{2}{7}\right), \left(0, 1, 1, \frac{4}{7}\right) \right\}.$$

Sol 10 Given system $2x_1 - x_2 = h$ — ①

$-6x_1 + 3x_2 = k$. — ②

Multiply ① × 3: we get $6x_1 - 3x_2 = 3h$ — ③

Now add ② and ③ we get

$$0 = 3h + k.$$

If $k+3h$ is non-zero then the system has no solutions.

Therefore the system is consistent for any values of h and k that make $k+3h=0$.

sol 5:

since the coefficient matrix has four pivots, there is a pivot in every row of the coefficient matrix. This means that when the coefficient matrix is row reduced, it will not have a row of zeros, thus the corresponding row reduced augmented matrix can never have a row of the form $[0\ 0 \ldots 0\ b]$ where b is a non-zero number. Therefore the system is consistent. Moreover since there are seven columns in the coefficient matrix and only four pivot columns, there will be three free variables resulting in infinite many solutions.

Sol 8:

The augmented matrix is $\begin{bmatrix} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 4 & -8 & 12 & 1 \end{bmatrix}$;

Operation 1: $R_1 \leftrightarrow R_2$ (Interchange rows 1 and 2

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 4 & -8 & 12 & 1 \end{bmatrix}$$

Operation 2: $R_3 \to R_3 - 2R_1$

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & -2 & 8 & -1 \end{bmatrix}$$

Operation 3: $R_3 \to R_3 + 2R_2$

$$\begin{bmatrix} 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 8 \\ 0 & 0 & 0 & 15 \end{bmatrix}$$

We get system

$$\left.\begin{aligned} 2x_1 - 3x_2 + 2x_3 &= 1 \\ x_2 - 4x_3 &= 8 \\ 0 &= 15 \quad \text{———} \ (*) \end{aligned}\right\} \text{———} \ ①$$

(*) is never true.

Since system ① and system of equations given in the question have the same solution set,

$\therefore$ The original system is inconsistent.

Sol^n 6 We have by a Theorem;

Let $A \in M_{m\times n}(F)$ and suppose B is obtained from A by performing an elementary row [column] operation. Then there exists an $m\times m$ [$n \times n$] elementary matrix E such that $B = EA$ [$B = AE$]. In fact E is obtained from I_m [I_n] by performing the same elementary row [column] operation as that which was performed on A to obtain B.

Here Given matrix $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

Applying $R_1 \to R_1 - R_2$, we get

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Applying elementary row operation $R_2 \to R_2 + 2R_3$ on Identity matrix we get matrix B.

Now $R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Similarly applying elementary row operation $R_1 \to R_1 + R_2$ on Identity matrix we get

$$A' = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Hence;

$$A = A'B$$

where A' and B are elementary matrices.

Sol 9

Given system in augmented form

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ -1 & -3 & 0 & 4 \end{bmatrix}$$

$$R_3 \longrightarrow R_3 + R_1$$

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 1 & 8 \end{bmatrix}$$

Now $\quad R_3 \longrightarrow R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

we have last row as $[0 \, - - \, 0 \;\, 9]$

Hence the system is inconsistent.

Therefore given three lines do ~~to~~ not have a common point of intersection.

Sol 7

Corollary 3, sec 3.2, ch-3 Friedberg.

Sol 11.

(a) we have $\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix}$

$R_2 \to R_2 + 2R_1$

we get $\begin{bmatrix} 1 & h & -3 \\ 0 & 4+2h & 0 \end{bmatrix}$

so, we don't have any row of the form $[0 \ \ 0 \ \ 0 . b]$ where b is non-zero number,

hence system is consistent.

Unique if $h \neq -2$ otherwise infinite many.

(b) We have $\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix}$

$R_2 \to R_2 + 4R_1$

$\begin{bmatrix} 1 & 3 & -2 \\ 0 & h+12 & 0 \end{bmatrix}$

Again it is consistent.

similarly by part (a).

Sol^n 12

To find inverse of A.

To use elementary row operations to transform

$$(A|I) = \left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 2 & 4 & 5 & 0 & 1 & 0 \\ 3 & -5 & 6 & 0 & 0 & 1 \end{array}\right]$$

$$R_2 \to R_2 - 2R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 0 & -1 & -2 & 1 & 0 \\ 3 & -5 & 6 & 0 & 0 & 1 \end{array}\right]$$

Interchanging R_2 and R_3 $(R_2 \leftrightarrow R_3)$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 3 & -5 & 6 & 0 & 0 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right]$$

$$R_2 \to R_2 - 3R_1$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -11 & -3 & -3 & 0 & 1 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right]$$

$$R_2 \to -\frac{1}{11} R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 3/11 & 3/11 & 0 & -1/11 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right]$$

$$R_1 \to R_1 - 2R_2$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 27/11 & 5/11 & 0 & 2/11 \\ 0 & 1 & 3/11 & 3/11 & 0 & -1/11 \\ 0 & 0 & -1 & -2 & 1 & 0 \end{array}\right]$$

$$R_3 \longrightarrow -R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 27/11 & \frac{5}{11} & 0 & \frac{2}{11} \\ 0 & 1 & 3/11 & 3/11 & 0 & -1/11 \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

$$R_1 \longrightarrow R_1 - \frac{27}{11} R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-49}{11} & \frac{27}{11} & \frac{2}{11} \\ 0 & 1 & \frac{3}{11} & \frac{3}{11} & 0 & \frac{-1}{11} \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

$$R_2 \longrightarrow R_2 - \frac{3}{11} R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{-49}{11} & \frac{27}{11} & \frac{2}{11} \\ 0 & 1 & 0 & \frac{-3}{11} & \frac{3}{11} & \frac{-1}{11} \\ 0 & 0 & 1 & 2 & -1 & 0 \end{array}\right]$$

Hence, Inverse is $\begin{bmatrix} \frac{-49}{11} & \frac{27}{11} & \frac{2}{11} \\ \frac{-3}{11} & \frac{3}{11} & \frac{-1}{11} \\ 2 & -1 & 0 \end{bmatrix}$

$$\therefore \quad X = \begin{bmatrix} \frac{-49}{11} & \frac{27}{11} & \frac{2}{11} \\ \frac{-3}{11} & \frac{3}{11} & \frac{-1}{11} \\ 2 & -1 & 0 \end{bmatrix}$$

Sol^n 13

We have to check whether the augmented system is consistent or not.

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{array}\right]$$

$$R_3 \rightarrow R_3 + 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{array}\right]$$

Last row can never be true for any variable.

Therefore the system is inconsistent.

Hence, we can't write b as a linear combination of the vectors formed from the columns of A.

Sol^n 14

(i) Given matrix $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 2 \\ 1 & -4 & 7 \end{bmatrix}$

Solⁿ 14

(i) Given matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$

Consider $(A \mid I)$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 1 & 1 & -1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -1 & -4 & -1 & 1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right]$$

$R_2 \to -R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 4 & 1 & -1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - 2R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5 & -1 & 2 & 0 \\ 0 & 1 & 4 & 1 & -1 & 0 \\ 0 & 0 & -2 & -1 & 0 & 1 \end{array}\right]$$

$R_3 \to -\frac{1}{2} R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -5 & -1 & 2 & 0 \\ 0 & 1 & 4 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1/2 & 0 & -1/2 \end{array}\right]$$

$$R_2 \longrightarrow R_2 - 4R_3$$
$$R_1 \longrightarrow R_1 + 5R_3$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3/2 & 2 & -5/2 \\ 0 & 1 & 0 & -1 & -1 & 2 \\ 0 & 0 & 1 & 1/2 & 0 & -1/2 \end{array}\right].$$

Hence Inverse of $A = \begin{bmatrix} 3/2 & 2 & -5/2 \\ -1 & -1 & 2 \\ 1/2 & 0 & -1/2 \end{bmatrix}$

Now we have system

$$Ax = b$$

where $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{bmatrix}$, $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$

Multiplying above equation by A^{-1} we get

$$A^{-1}Ax = A^{-1}b$$

$$\Rightarrow \quad Ix = A^{-1}b$$

$$\Rightarrow \quad x = A^{-1}b.$$

$$\therefore \quad x = \begin{bmatrix} \frac{3}{2} & 2 & -5/2 \\ -1 & -1 & 2 \\ \frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$

$$x = \begin{bmatrix} -6 \\ 5 \\ -1 \end{bmatrix}$$

$$\Rightarrow \quad x_1 = -6, \quad x_2 = 5 \quad \text{and} \quad x_3 = -1$$

(ii) Given matrix $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

Consider $(A \mid I)$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 2 & 5 & 0 & 1 \end{array}\right]$$

$$R_2 \to R_2 - 2R_1$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & -1 & -2 & 1 \end{array}\right]$$

$$R_2 \to -R_2$$

$$\left[\begin{array}{cc|cc} 1 & 3 & 1 & 0 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

$$R_1 \to R_1 - 3R_2$$

$$\left[\begin{array}{cc|cc} 1 & 0 & -5 & 3 \\ 0 & 1 & 2 & -1 \end{array}\right]$$

Hence $\quad A^{-1} = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}$

Now we have system $\quad Ax = b$

$$A^{-1}Ax = A^{-1}b$$

$$\Rightarrow \quad x = A^{-1}b$$

$$\Rightarrow \quad x = \begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad x = \begin{bmatrix} -11 \\ 5 \end{bmatrix}$$

Sol^n 15

(a) Consider augmented matrix

$$\left[\begin{array}{ccc|c} 3 & -4 & 2 & 0 \\ -9 & 12 & -6 & 0 \\ -6 & 8 & -4 & 0 \end{array}\right]$$

Changes in RHS vector will not affect it

Hence, consider

$$\begin{bmatrix} 3 & -4 & 2 \\ -9 & 12 & -6 \\ -6 & 8 & -4 \end{bmatrix}$$

$R_2 \rightarrow R_2 + 3R_1$

$R_3 \rightarrow R_3 + 2R_1$

$$\begin{bmatrix} 3 & -4 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

We have only one equation

$$3x_1 - 4x_2 + 2x_3 = 0.$$

$$x_1 = \frac{4}{3}x_2 - \frac{2}{3}x_3$$

Here x_2 and x_3 are free variables

can take any value.

Let $x_2 = s$, $x_3 = t$

$$\therefore \quad x_1 = \frac{4}{3}s - \frac{2}{3}t$$

We have solution set

$$\left\{ \begin{bmatrix} \frac{4}{3}s - \frac{2}{3}t \\ s \\ t \end{bmatrix} : s, t \in \mathbb{R} \right\}$$

(b) Consider augmented matrix

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{array}\right]$$

$$R_2 \to R_2 - 3R_1$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -1 & 3 \\ 0 & 0 & 1 & -7 \end{array}\right]$$

So we have

$$x_1 - 2x_2 - x_3 = 3$$

$$x_3 = -7$$

Hence $x_1 - 2x_2 = -4$

$$x_2 = \frac{4 + x_1}{2}.$$

Hence solution set

$$\left\{ \begin{bmatrix} s \\ \frac{4+s}{2} \\ -7 \end{bmatrix} : s \in \mathbb{R} \right\}.$$