MA 102 : Linear Algebra and Integral Transforms
Tutorial Sheet - 3
Second Semester of Academic Year 2019-2020

1. Row reduce the matrix A to echelon form and locate the pivot columns.

$$A = \begin{bmatrix} 0 & 3 & -6 & 6 & 4 & -5 \\ 3 & -7 & 8 & -5 & 8 & 9 \\ 3 & -9 & 12 & -9 & 6 & 15 \end{bmatrix}$$

2. Find the row reduced echelon form of the following matrix:

$$A = \begin{bmatrix} 1 & 2 & -3 & 1 & 2 \\ 2 & 4 & -4 & 6 & 10 \\ 3 & 6 & -6 & 9 & 13 \end{bmatrix}$$

3. Find the dimension of the row space and column space of the following matrix:

$$A = \begin{bmatrix} 1 & -3 & 4 & -2 & 5 & 4 \\ 2 & -6 & 9 & -1 & 8 & 2 \\ 2 & -6 & 9 & -1 & 9 & 7 \\ -1 & 3 & -4 & 2 & -5 & -4 \end{bmatrix}$$

4. For the given matrix

$$A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$$

   (a) Find a basis for the null space.
   (b) Find a basis for the column space.
   (c) Find a basis for the row space by reducing the matrix to row reduced echelon form.

5. Suppose a 4 x 7 coefficient matrix for a system of equations has 4 pivots. Is the system consistent? If the system is consistent, how many solutions are there?

6. Express the given invertible matrix as a product of elementary matrices.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

7. Show that every non-singular matrix is a product of elementary matrices.

8. Determine if the following system is consistent:

$$\begin{aligned} x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 4x_1 - 8x_2 + 12x_3 &= 1 \end{aligned}$$

9. Do the three planes $x_1 + 2x_2 + x_3 = 4, x_2 - x_3 = 1$ and $-x_1 - 3x_2 = 4$ have a common point of intersection? Explain.

10. For what values of h and k is the following system consistent?

$$\begin{aligned} 2x_1 - x_2 &= h \\ -6x_1 + 3x_2 &= k \end{aligned}$$

11. Determine the values of h such that the matrix is the augmented matrix of a consistent linear system.

(a) $\begin{bmatrix} 1 & h & -3 \\ -2 & 4 & 6 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 3 & -2 \\ -4 & h & 8 \end{bmatrix}$

12. Obtain a non-singular matrix X such that $XA = I$ where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & -5 & 6 \end{bmatrix}$$

13. Determine if b is a linear combination of the vectors formed from the columns of matrix A where

$$A = \begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 3 \\ -7 \\ -3 \end{bmatrix}.$$

14. For each system of linear equations with invertible coefficient matrix A

(a) Compute the inverse of A.

(b) Then use A^{-1} to solve the systems:

i. $$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 1 \\ x_1 + x_2 - x_3 &= 0 \\ x_1 + 2x_2 + x_3 &= 3 \end{aligned}$$

ii. $$\begin{aligned} x_1 + 3x_2 &= 4 \\ 2x_1 + 5x_2 &= 3 \end{aligned}$$

15. Find the solutions of the systems.

(a) $$\begin{aligned} 3x_1 - 4x_2 + 2x_3 &= 0 \\ -9x_1 + 12x_2 - 6x_3 &= 0 \\ -6x_1 + 8x_2 - 4x_3 &= 0 \end{aligned}$$

(b) $$\begin{aligned} x_1 - 2x_2 - x_3 &= 3 \\ 3x_1 - 6x_2 - 2x_3 &= 2 \end{aligned}$$

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