Syllabus for comprehensive Examination (written):

1. Algebra: Groups, Simple Groups, Group action, Solvable Groups, Nilpotent Groups, Simplicity of Alternating Groups, Composition Series, Jordon-Holder Theorem, Semi-direct product, Split extension, Free Group, Free Abelian Group, Free Product.
Rings, Examples (Including Polynomial Rings, Formal power Series Ring, Matrix Ring, Group Rings), Ideals, Prime and maximal Ideals, Rings of Fractions, Chinese Remainder Theorem for pairwise comaximal Ideals, Euclidean domains, Principal Ideal Domains and Unique Factorization Domain, Polynomial rings over UFD.
Fields, Prime Subfields, Field Extensions, Finite feild.
2. Real Analysis: Metric Spaces, Compactness, Perfect Sets, Connectedness, Completeness in Rn, Cantor set, Sequences and Series of functions, Uniform convergence and its relation to Continuity, Differentiation and Integration, Equicontinuous families of functions, Ascoli's theorem, Weierstrass approximation theorem. Derivative as a linear transformation, Contraction principle, Inverse and Implicit function theorems. Lebesgue Measure and Integration: Measure of open sets and compact sets, Inner and Outer measure, measurable sets and its properties, measurable functions, Lebesgue integral of a bounded function, the general Lebesgue integral, square integrable functions.
3. Differential equations: First and second order ordinary differential equations, existence and uniqueness theorems, Picard's method of successive approximations, dependence on initial conditions, Boundary value problems, Sturm-Liouville problems, Solution of ODE by Laplace Transform, Solution to the system of ordinary differential equations.

First order partial differential equation. Cauchy problem, partial differential equation of second and higher order, classification of second order equations, Canonical form, Laplace equation, Diffusion equation, Wave equation, Variable separable method, Fourier Transform and its applications.
4. Numerical Analysis : Introduction to error, Root finding of non-linear equations. Systems of equations: Direct and indirect methods. Interpolation. Numerical differentiation. Numerical integration: Trapezoidal and Simpsons rules, Newton-Cotes formula, Method of undermind coefficients and Gaussian Quadrature. Numerical solution of ordinary differential equations: Initial value problems: Numerical stability, Taylor series method, Euler and modified Euler methods and stability analysis, Runge-Kutta methods, Multistep methods, Predictor- Corrector method.

5. Linear Algebra: Vector spaces over fields, subspaces, bases and dimensions; Systems of linear equations, matrices, rank, Gaussian elimination; Linear transformations, representation of linear transformations by matrices, rank-nullity theorem, change of basis, dual spaces, transposes of linear transformations; Eigen values and Eigen vectors, characteristic polynomials, minimal polynomials, Cayley-Hamilton Theorem, triangulation, diagonalization, Jordan canonical form, rational canonical form; Inner product spaces, Gram-Schmidt ortho-normalization, least square approximation, linear functionals and adjoints, Hermitian, self-adjoint, unitary and normal operators, Spectral Theorem for normal operators; Bilinear forms, symmetric and skew-symmetric bilinear forms, real quadratic forms, positive definiteness.

6. Probability & Stochastic Processes or Probability & Statistics:

6a. Probability & Stochastic Processes: Probability spaces, random variables, random vectors, probability distributions, independence, joint distributions, expectation, variance, convergence of sequence of random variables, law of large numbers, central limit theorem, moment generating function, characteristic function, definition and properties of conditional expectation.

Classification of stochastic processes, Bernoulli process, Poisson process, Markov chains, Kolmogorov equations, discrete time martingales, Doob's convergence theorem, Doob's decomposition of a stochastic process, L^(p) inequality, random walks.
Brownian motion, stopping times, continuous time martingales, recurrence of Brownian motion, Feynman-Kac formula, the Ito integral for Brownian motion, processes of bounded quadratic variation, applications.

6b. Probability & Statistics:

Probability spaces, random variables, random vectors, probability distributions, independence, joint distributions, expectation, variance, convergence of sequence of random variables, law of large numbers, central limit theorem, moment generating function, characteristic function, definition and properties of conditional expectation.

Random sampling, sample characteristics and their distributions, Chi-square, t and F distributions, exact sampling distributions, point estimation, sufficiency, completeness and ancillarity, unbiased estimation, lower bound for variance of an unbiased estimator, maximum likelihood estimation, method of moments, Bayes and minimax estimation, testing of hypotheses, Neyman-Pearson lemma, confidence estimation, methods for findingconfidence intervals.